Week 6: Domain coupling, removing transformers, Norton's theorem

Microphone and Loudspeaker Design - Level 5

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What are we covering today?

1. Domain coupling
2. Acoustic loading
3. Removing transformers
4. Norton and Thevenin's theorems
5. Tutorial questions

A weekly fact about Salford..!

Did you know...

- One of Salford's oldest buildings is the Grade I listed Ordsall Hall, a Tudor mansion and former stately home in nearby Ordsall. It dates back over 750 years. Apparently is proper haunted...

Domain coupling

- Transducers convert one form of energy to another
  - two important types:
    1. Electro-mechanic: electrical to mechanical
    2. Mechano-acoustic: mechanical to acoustical

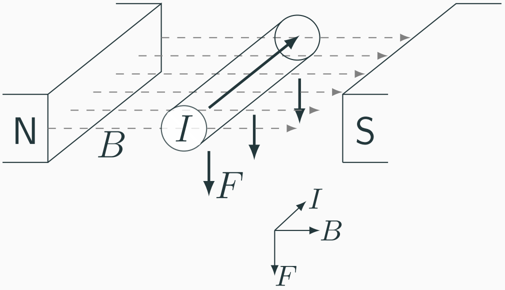


Figure 1: Lorentz force on a charge carrying conductor

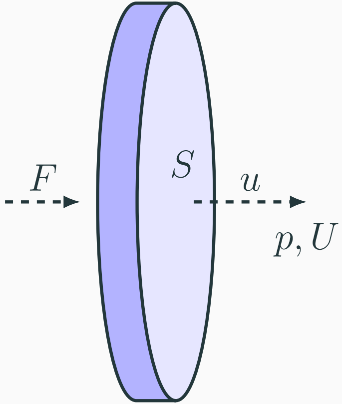


Figure 2: Mechano-acoustic transduction

Transducers: mechano-acoustic

- The equations that couple the mechanical and acoustic domains are:

$$F = pS \tag{1}$$

$$U = uS \tag{2}$$

- The first relates pressure to force
- The second relates volume velocity to surface velocity

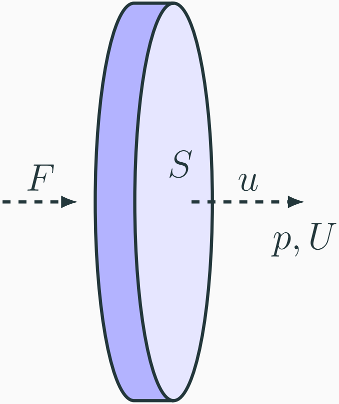


Figure 2: Mechano-acoustic transduction

- The Lorentz force for a coil of wire of length L in a magnetic field of strength B through which a current I passes, is given by

$$F = BLI \tag{3}$$

- But, electro-dynamic transduction is a two way phenomena...
- When a conductor moves in a magnetic field, a voltage is generated across its length

$$V = BLu \tag{4}$$

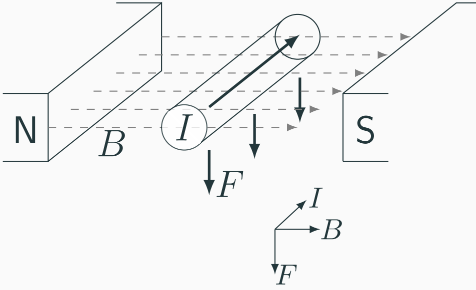


Figure 3: Lorentz force on a charge carrying conductor

Transducers: summary

- Electro-mechanical transduction:

$$F = BLI \tag{5}$$

$$u = \frac{V}{BL} \tag{6}$$

- Mechano-acoustical transduction:

$$p = \frac{F}{S} \tag{7}$$

$$U = Su \tag{8}$$

- The equations happen to look very similar to the equations of an ideal transformer...

Ideal transformers

- The turns ratio α determines voltage change

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \alpha \qquad (9)$$

- Reciprocal turns ratio determines effect on current

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{\alpha} \qquad (10)$$

- Gives us two equations:

$$V_p = \alpha V_s \qquad I_p = \frac{1}{\alpha} I_s \qquad (11)$$

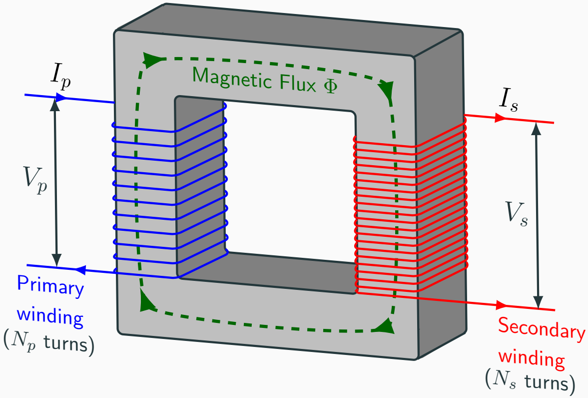


Figure 4: Ideal transformer

- Ideal transformer

$$V_p = \alpha V_s \qquad I_p = \frac{1}{\alpha} I_s \qquad (12)$$

- Electro-mechanic transduction

$$V = BLu \qquad I = \frac{1}{BL} F \qquad (13)$$

- Mechano-acoustic transduction

$$u = \frac{1}{S} U \qquad F = Sp \qquad (14)$$

- Look pretty similar don't they... $\alpha = ??$

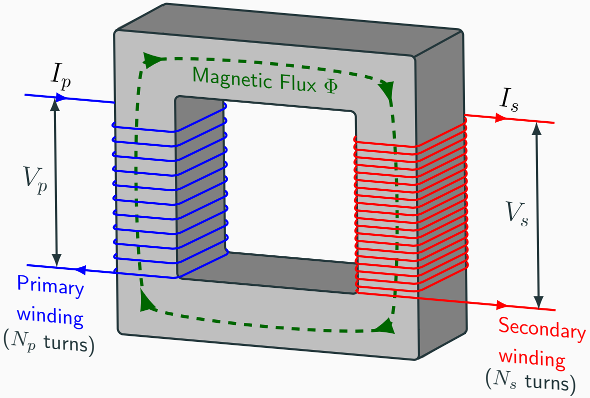


Figure 4: Ideal transformer

Domain coupling by ideal transformers: electro-mechanical

- Ideal transformer with turns ratio: $\alpha = BL$ vs. electro-mechanical transduction:

$$V_p = \alpha V_s \longleftrightarrow V = BLu \qquad I_p = \frac{1}{\alpha} I_s \longleftrightarrow I = \frac{1}{BL} F \tag{15}$$

- Secondary side: $u \sim V$, $F \sim I$ (what analogy is this?)

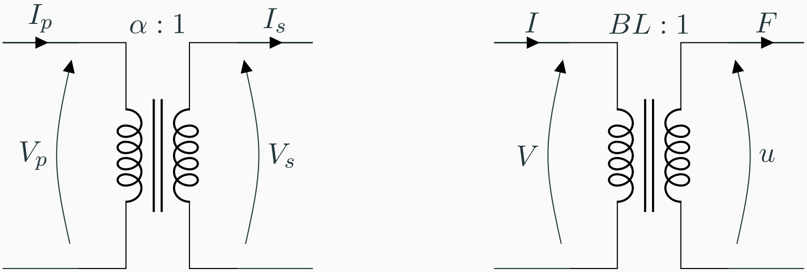


Figure 5: Ideal transformer vs. electro-mechanical coupling.

Domain coupling by ideal transformers: electro-mechanical

- Secondary side: $u \sim V$, $F \sim I$ - **mobility analogy**
- Substitute in our equivalent mobility-based circuit for a mass-spring-damper
- Resistor and inductor model the properties of the voice coil

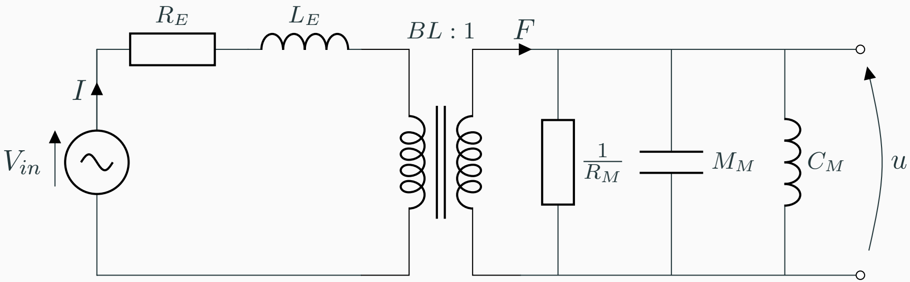


Figure 6: Ideal transformer coupling between electrical and mechanical domains.

Domain coupling by ideal transformers: mechano-acoustical

- Ideal transformer vs. mechano-acoustical transduction with turns ratio: $\alpha = 1/S$

$$V_p = \alpha V_s \longleftrightarrow u = \frac{1}{S}U \qquad I_p = \frac{1}{\alpha}I_s \longleftrightarrow F = Sp \tag{16}$$

- Secondary side: $U \sim V$, $p \sim I$ (what analogy is this?)

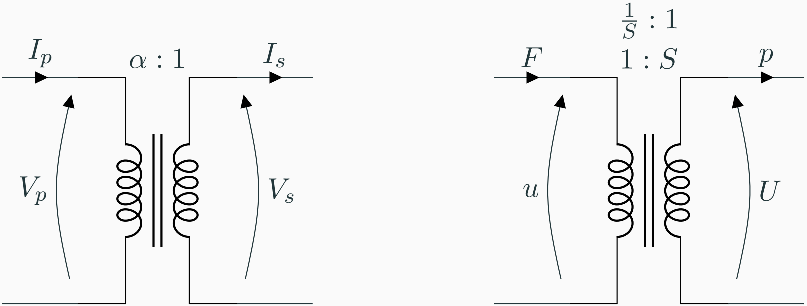


Figure 7: Ideal transformer vs. electro-mechanical coupling.

Domain coupling by ideal transformers: mechano-acoustical

- Secondary side: $U \sim V$, $p \sim I$ - **mobility analogy**
- The loudspeaker driver is currently modelled in a vacuum..!

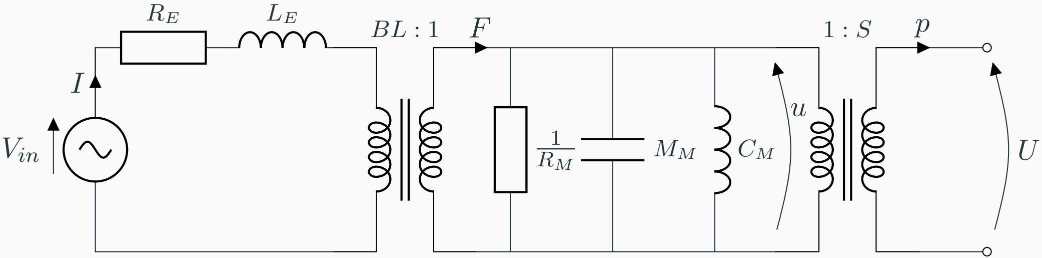


Figure 8: Ideal transformer coupling between electrical and mechanical domains.

Acoustic loading

Acoustic loading

- After the mechano-acoustic transformer the acoustic quantities follow the **mobility** analogue: $V \rightarrow U$, $I \rightarrow p$
- If a loudspeaker is in motion it is impeded by the air loading on the front and rear of the diaphragm.
- This loading will depend on the type of enclosure and the type of diaphragm - we can define them generally as the front and back impedances Z_{Af} and Z_{Ab}
- These have the same velocity due to the diaphragms motion, so the electrical components must share the same voltage since $U \rightarrow V$...
- This means the acoustic impedances **must be in parallel**!

Acoustic loading

- This equivalent circuit represents a complete general model of a loudspeaker.
- But what are Z_{Af} and Z_{Ab}? Their form will depend on the type of loudspeaker considered.

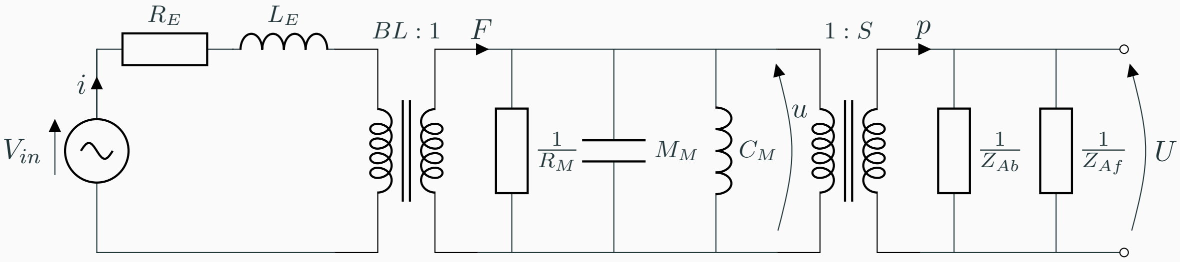


Figure 9: Equivalent circuit of a loudspeaker.

Acoustic loading

- The acoustic domain (Z_{Af} and Z_{Ab}) are mostly dominated by the design of a loudspeakers enclosure.
- Many different types of enclosures, all have their own acoustic characteristics
- Over next few weeks we will focus on:

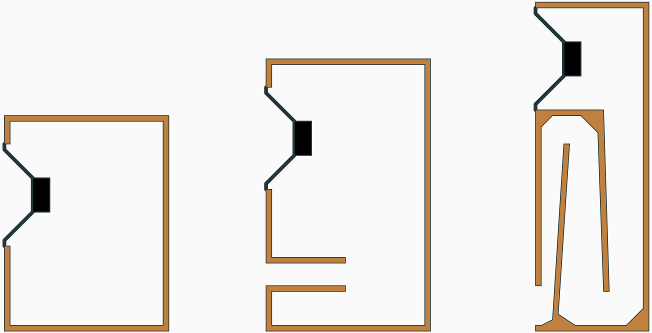
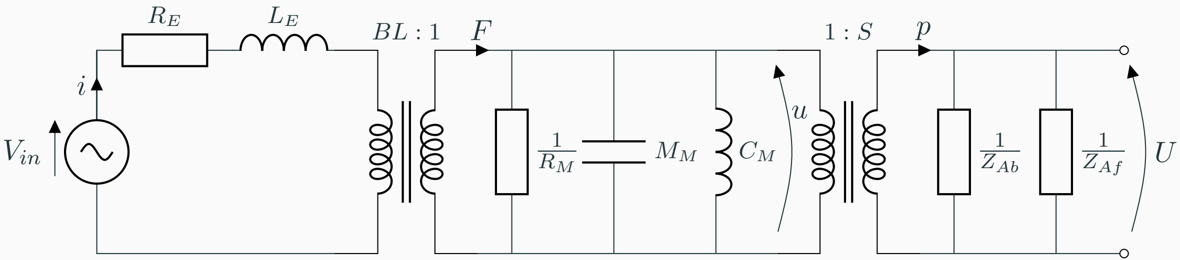


Figure 10: Typical loudspeaker systems.

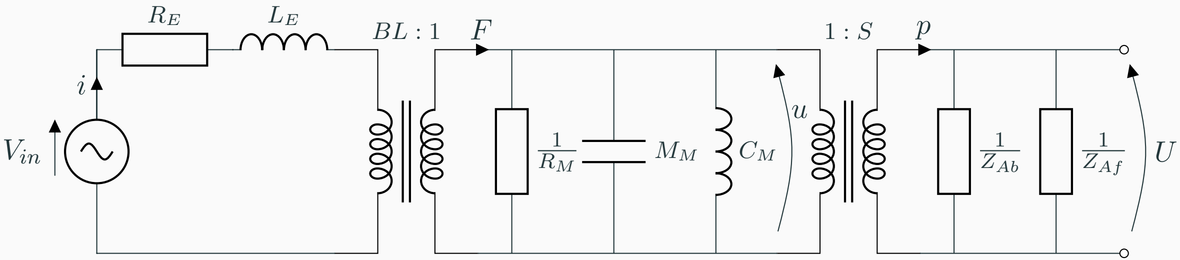
Removing transformers

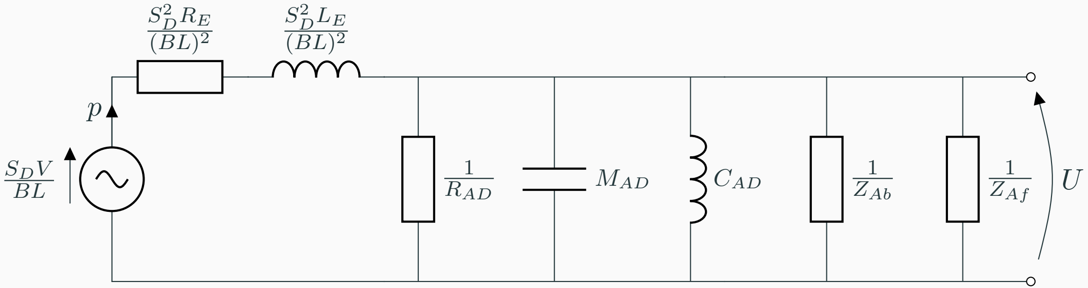
What are we aiming for?



- This is a complete equivalent circuit for a loudspeaker - but it is quite awkward to analyse (because of mixed series/parallel components, transformers).
- We want to simplify the circuit. First we remove the transformers...

What are we aiming for?





Removing transformers: left to right

- We want everything in terms of V_2 and I_2 (**move components to RHS**)
- Moving onto secondary winding side:
  - Scale impedances by reciprocal of turns ratio squared
  - Scale voltage source by reciprocal of turns ratio
  - Scale current sources by turns ratio

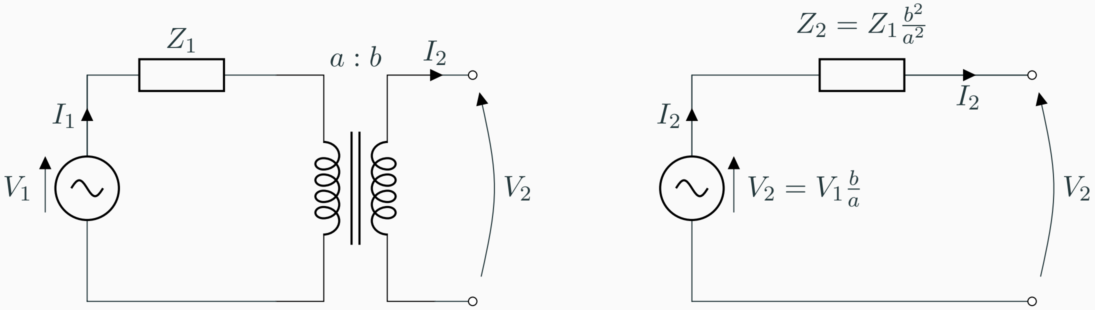


Figure 11: Moving electrical components across a transformer - left to right

Removing transformers: right to left

- We want everything in terms of V_1 and I_1 (**move components to LHS**)
- Moving onto primary winding side:
  - Scale impedances by turns ratio squared
  - Scale voltage source by turns ratio
  - Scale current sources by reciprocal of turns ratio

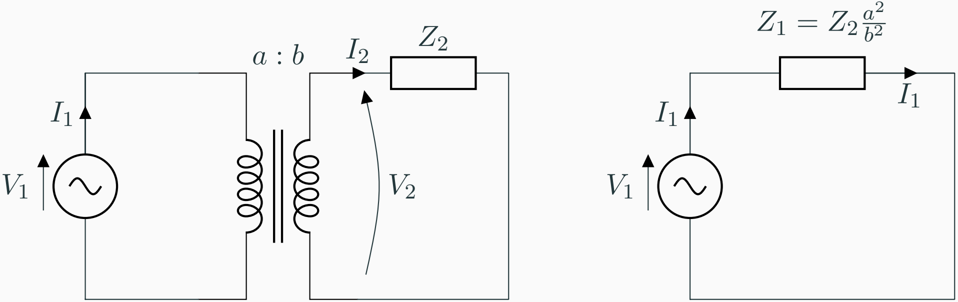
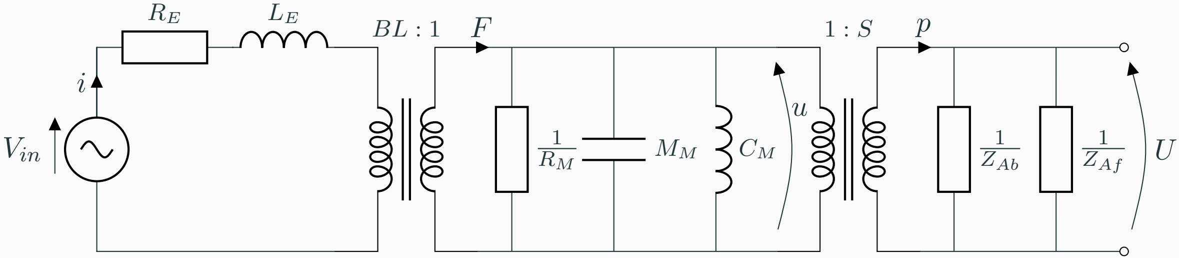
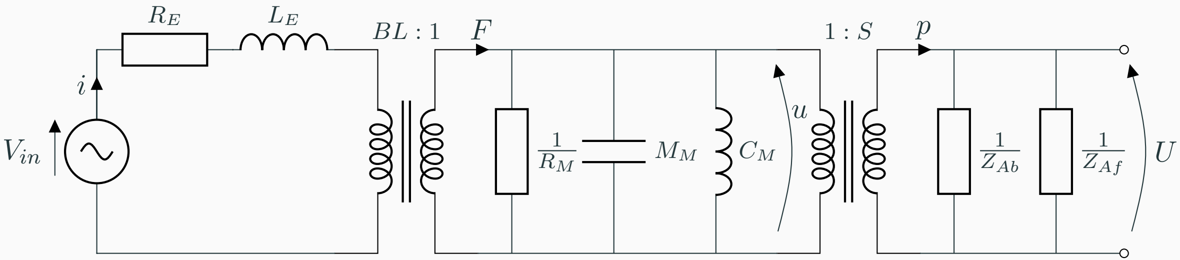


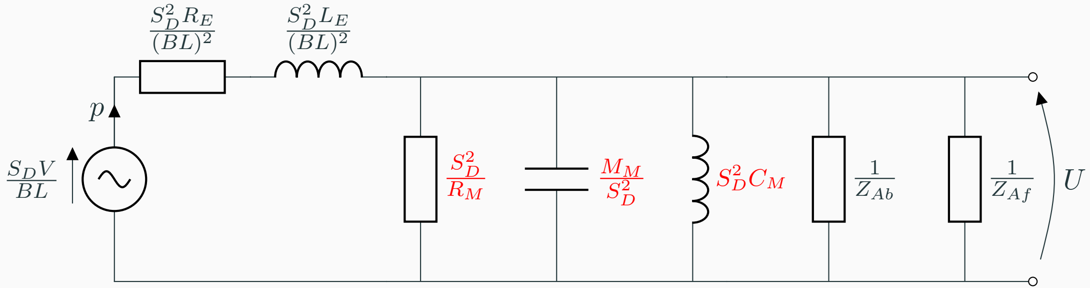
Figure 12: Moving electrical components across a transformer - left to right

Removing transformers: equivalent circuit



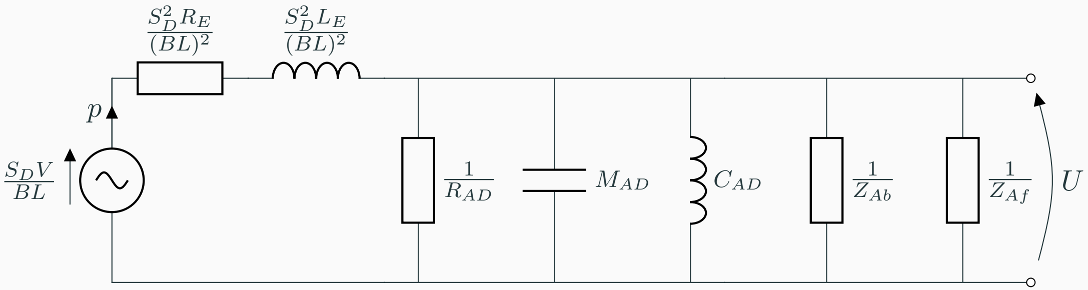
Removing transformers: equivalent circuit





Removing transformers: equivalent circuit

- Use acoustic mass, stiffness and damping: M_{AD}, C_{AD}, R_{AD}
  - Subscripts: A denotes *acoustic* domain, D denotes *driver* property
- Don't really like everything being in parallel. Makes calculating the impedance a bit more tricky...



Norton and Thevenin's theorems

Norton's theorem

- Norton's theorem states that '*any linear electrical network that contains only voltage sources, current sources and impedances can be replaced by an ideal current source* I_{no} *in parallel with an impedance* Z_{no}'.

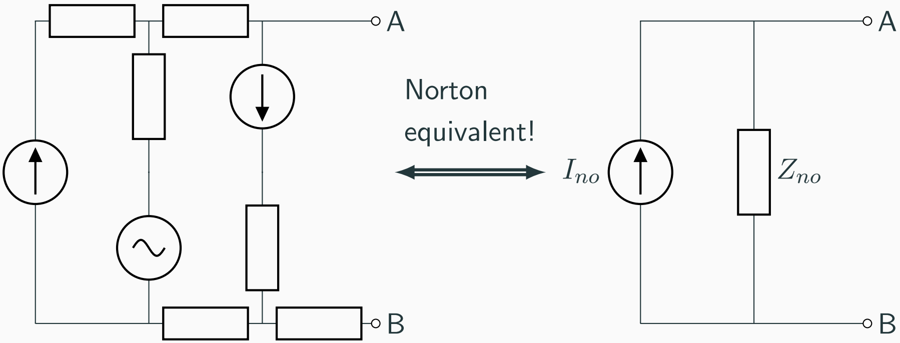


Figure 13: Norton equivalent circuit.

Thevenin's theorem

- Thevenin's theorem states that '*any linear electrical network that contains only voltage sources, current sources and impedances can be replaced by an equivalent combination of an ideal voltage source* V_{th} *in series with an impedance* Z_{th}.'

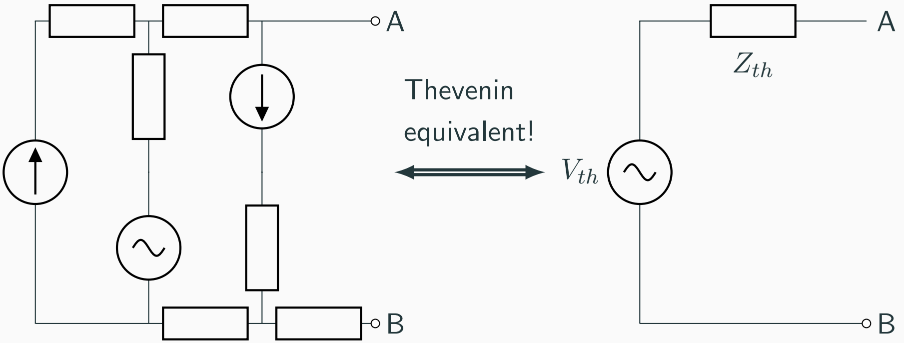


Figure 14: Thevenin equivalent circuit.

Equivalent loudspeaker model: application of Norton's theorem

- The procedure for applying Norton's theorem is as follows:
  - The equivalent current I_{no} is the current obtained at terminals A B of the network with terminals A B short circuited.
  - The equivalent impedance Z_{no} is the impedance obtained at terminals A B of the network with all its voltage sources short circuited and all its current sources open circuited.

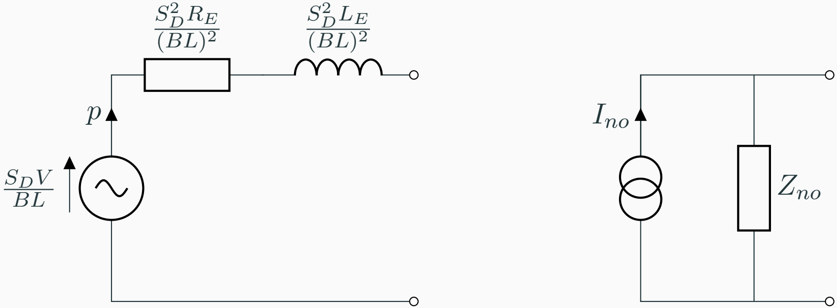


Figure 15: Equivalent Norton circuit for electrical components in loudspeaker model.

Equivalent loudspeaker model: application of Norton's theorem

- Using Norton's theorem we replace a electrical part (voice coil resistance and inductance) with the equivalent Norton current source and impedance

$$I_{no} = \frac{VBL}{S_D(R_E + j\omega L_E)} \qquad Z_{no} = \frac{R_E S_D^2}{(BL)^2} + \frac{j\omega L_E S_D^2}{(BL)^2} \tag{17}$$

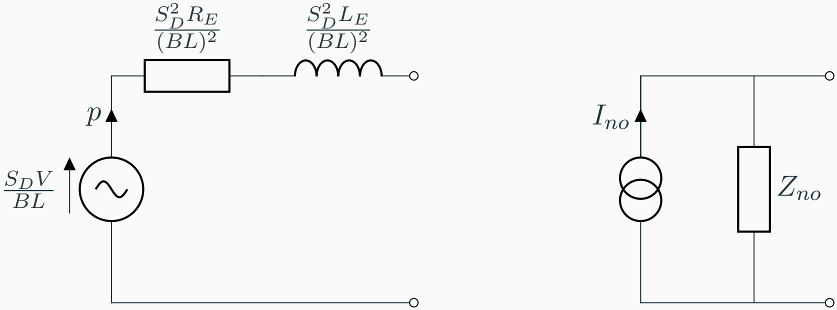
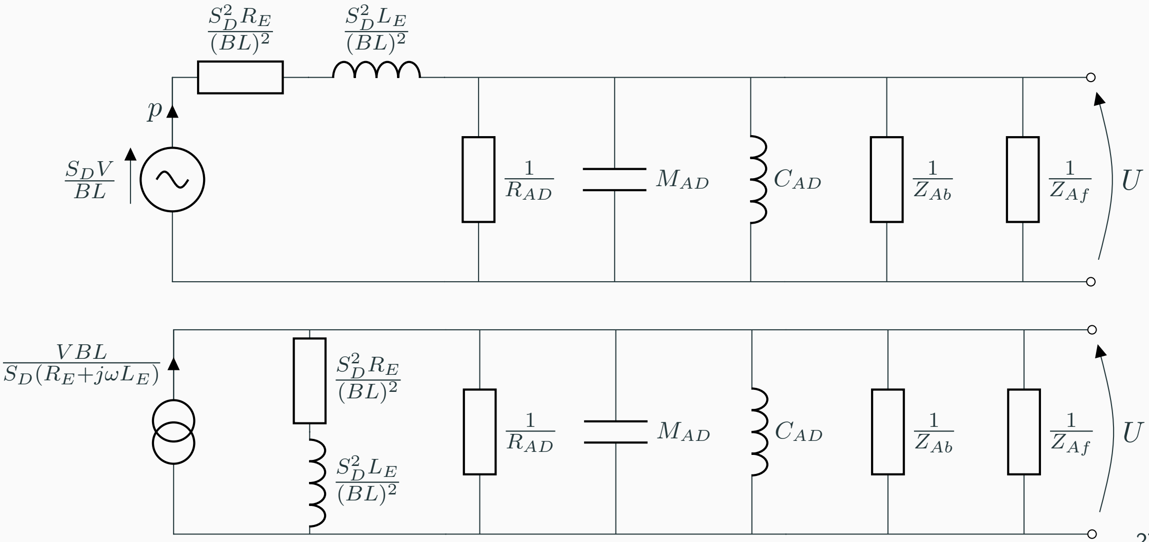


Figure 15: Equivalent Norton circuit for electrical components in loudspeaker model.

Equivalent loudspeaker model: application of Norton's theorem



Equivalent loudspeaker model: taking the dual

- We have a parallel equivalent (mobility) circuit, take its dual to obtain a series (impedance) circuit:
  1) Current source $\leftrightarrow$ voltage source (and vice versa)
  2) Capacitor $\leftrightarrow$ inductor (and vice versa)
  3) Resistor $\leftrightarrow$ conductor (1/resistor) (and vice versa)
  4) Series $\leftrightarrow$ parallel (and vice versa)

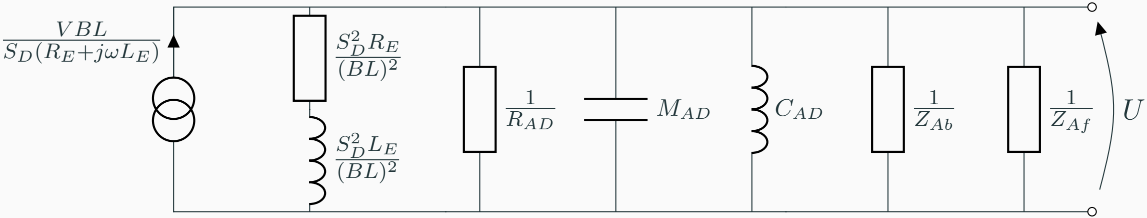


Figure 16: Equiv. mobility circuit with transformers removed and Norton's theorem applied.

Our finished circuit (for now...)

- This equivalent circuit represents a complete low frequency (lumped parameter) model of a loudspeaker driver loaded by two arbitrary acoustic impedances.
- The acoustic loading Z_{Af} and Z_{Ab} dictates whether the driver is housed in an infinite baffle, sealed or vented cabinet.

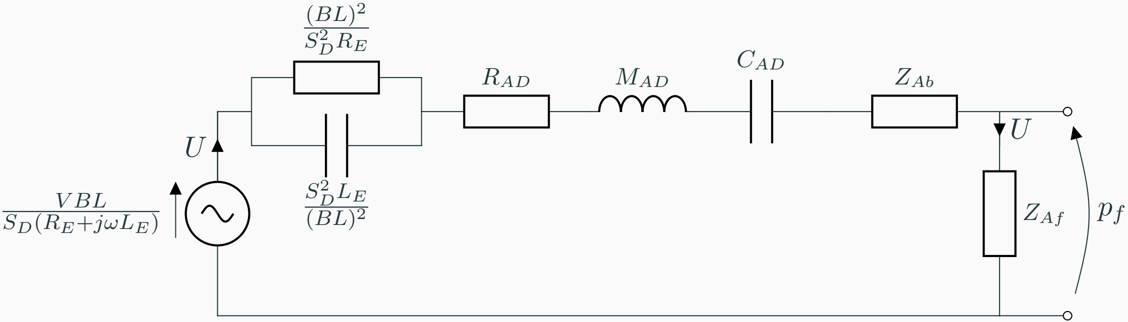


Figure 17: Equiv. impedance circuit with transformers removed and Norton's theorem applied.

Next week...

- Acoustic radiation (monopole, dipole, piston)
- Radiation impedance
- Infinite baffle loudspeaker

- Reading:
  - Sound radiation: lecture notes Ch. 7 (all)
  - Infinite baffle loudspeaker: lecture notes Sec. 8.1